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## MAGNETIC CONVECTION IN A NONUNIFORMLY ROTATING LAYER OF CONDUCTIVE NANOFLUID IN AN EXTERNAL HELICAL MAGNETIC FIELD

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Investigating the effects of Brownian diffusion and thermophoresis of nanoparticles on the forced convection of nanofluids in heat exchangers is a critical and relevant area of research. Many researchers [1] have investigated convective instability in nanofluids, considering the effects of Brownian diffusion and thermophoresis. There is currently a gap in the literature regarding magnetic convection in nonuniformly rotating media. This issue has been partially addressed in articles [2-4], focusing on various astrophysical applications. Paper [5] explored the idea of using metallic nanofluids for laboratory simulations of magnetorotational instability. The study specifically examined the influence of the Hall current on thermal instability in a nonuniformly rotating nanofluid layer subjected to a constant magnetic field. Using the local WKB approximation, in [6] was derived a dispersion equation describing the evolution of small perturbations in a thin nanofluid layer. Specifically, it was obtained a dispersion equation for magnetorotational instability (MRI) in the absence of a temperature gradient but with a gradient in nanoparticle concentration. For large Hall parameter values ( $M \gg 1$ ), it was determined an expression for the critical Rossby number ( $Ro_{cr}$ ), which defines the threshold for nonuniform rotation profiles ( $Ro$ ). The investigation of paper [5] led to the following conclusions:

1. A negative rotation profile ( $Ro < 0$ ) lowers the threshold for the onset of convective instability.

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2. An increase in the Hall parameter ( $M$ ) can either delay or enhance the onset of convective instability, depending on the specific conditions.

3. Strengthening the axial magnetic field slows the onset of convection.

4. For a negative rotation profile ( $Ro < 0$ ), increasing the rotation parameter ( $Ta$ ) destabilizes convective instability.

5. Higher nanoparticle concentrations reduce the stationary critical Rayleigh number, promoting the onset of convective instability.

In another study [6], the criterion for the onset of convection was investigated in a Darcy-Brinkman porous medium layer saturated with an electrically conductive nanofluid under the influence of a helical magnetic field and uniform rotation. The model incorporates the effects of Brownian motion and thermophoresis for nanofluids, while the Darcy-Brinkman framework describes the porous medium. Instead of specifying the nanoparticle volume fraction at the boundaries, a zero nanoparticle flux boundary condition was adopted. In the absence of a temperature gradient, a novel type of instability was analyzed in a helical magnetic field within a thin nanofluid layer. The growth rate and the instability region were numerically determined as functions of the azimuthal magnetic field profile (magnetic Rossby number ( $Rb$ ) and the radial wave number ( $k$ )). In the presence of a temperature gradient, the stationary regime of nonuniformly rotating magnetoconvection was studied. Using linear stability theory, an exact analytical expression for the critical Rayleigh-Darcy number was derived, expressed in terms of various dimensionless parameters. The findings indicate that rotation and the axial (vertical) component of the helical magnetic field delay the onset of convection, whereas the azimuthal component destabilizes the system at positive magnetic Rossby numbers ( $Rb > 0$ ). Conditions for the stabilization and destabilization of stationary convection in a helical magnetic field were identified for metal oxide, metallic, and semiconductor nanofluids.

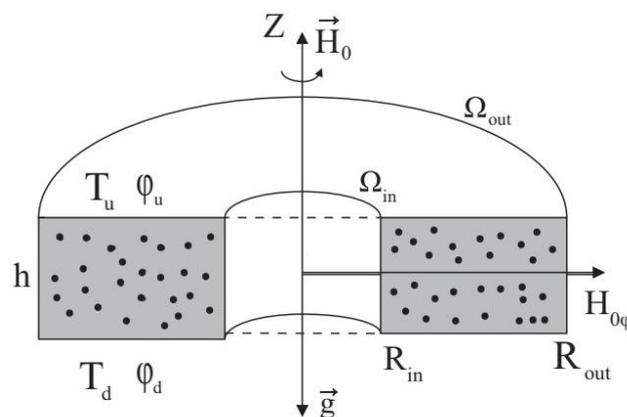


Fig. 1. **Geometry of the problem**

The intriguing effects observed in the thermal instability of thin-layer nanofluids under nonuniform rotation and an external helical magnetic field served as the primary motivation for this study. Consider a thin layer of an incompressible, viscous, electrically conducting nanofluid of thickness  $h$ , confined between two rotating cylinders with inner radius ( $R_{in}$ ) and outer radius ( $R_{out}$ ), such that  $h \ll (R_{out} - R_{in})$ . The nanofluid is bounded by two parallel planes located at  $z = 0$  and  $z = h$ , where the temperature  $T$  and the nanoparticle volume fraction  $\phi$  are held constant:  $T = T_d, \phi = \phi_d, T = T_u, \phi = \phi_u$ , with  $T_d > T_u, \phi_u > \phi_d$  (see Figure 1). The system is subjected to a constant gravitational field acting vertically downward along the  $z$ -axis. The electrically conducting nanofluid rotates with an angular velocity  $\Omega$  directed vertically upward along the  $z$ -axis. This rotation induces a steady azimuthal flow described by  $V_\theta = e_\phi \Omega(R)R$ , where  $\Omega(R)$  is the angular velocity as a function of the radial coordinate  $R$ . Additionally, the nanofluid is immersed in a helical magnetic field  $H_0$ , which consists of a nonuniform azimuthal component,  $H_{0\phi}$ , and a uniform axial component,  $H_{0z}$ . To describe the convective processes, we employ the Boussinesq-Oberbeck equations for hydrodynamics, tailored to an incompressible, electrically conducting nanofluid. These equations are formulated in a cylindrical coordinate system to account for the system's geometry. Using the local WKB approximation, we derived a dispersion equation for small perturbations in a thin nanofluid layer. Without a temperature gradient but with a nanoparticle concentration gradient:

- a) an axial magnetic field ( $H_{0z} \neq 0$ ) induces standard MRI,
- b) a nonuniform azimuthal field ( $H_{0\phi} \neq 0$ ) leads to azimuthal MRI,
- c) a helical field ( $H_0 = e_\phi H_{0\phi}(R) + e_z H_{0z}$ ) generates helical MRI.

We analyzed stationary convection modes in axial and helical magnetic fields, considering temperature and nanoparticle concentration gradients. The study focused on their dependence on nonuniform rotation profiles (Rossby number  $Ro$ ) and nonuniform azimuthal magnetic field profiles (magnetic Rossby number  $Rb$ ). We demonstrated that the presence of nanoparticles lowers the threshold value of the stationary critical Rayleigh number in both axial and helical magnetoconvection.

#### REFERENCES:

- [1] Nanofluid Dynamics and Transport Phenomenon (2024) Edited by Reshu Gupta, Mukesh Kumar Awasthi, Dhananjay Yadav, Yashvir Singh. Boca Raton, CRC Press, 264 p. <https://doi.org/10.1201/9781003527251>
- [2] Kopp, M. I., Tur, A. V., Yanovsky, V. V. (2019) Instabilities in the non-uniformly rotating medium with temperature stratification in the external uniform magnetic field, East European Journal of Physics, Vol. 1, 4-33. <https://doi.org/10.26565/2312-4334-2019-1-01>

**ABSCHNITT 19.**

PHYSIKALISCH UND MATHEMATISCH

- [3] Kopp, M. I., Tur, A. V., Yanovsky, V. V. (2020) Weakly nonlinear magnetic convection in a nonuniformly rotating electrically conductive medium under the action of modulation of external fields, East European Journal of Physics, Vol. 2, 5-37. <https://doi.org/10.26565/2312-4334-2020-2-01>
- [4] Kopp, M. I., Tur, A. V., Yanovsky, V. V. (2021) Magnetic convection in a nonuniformly rotating electrically conductive medium in an external spiral magnetic field, Fluid Dynamics Research, Vol. 53, 015509. <https://doi.org/10.1088/1873-7005/abd8dc>
- [5] Kopp, M. I., Yanovsky, V. V. (2022) Influence of the Hall current on the convective and magnetorotational instability in a thin layer of an electrically conductive nanofluid, Physics of Fluids, Vol. 34, 064107. <https://doi.org/10.1063/5.0094977>
- [6] Kopp, M. I., Yanovsky, V. V. (2022) Thermal convection in a rotating porous medium layer saturated by a nanofluid under a helical magnetic field, Journal of Applied Physics, Vol. 132, 084302. <https://doi.org/10.1063/5.0100131>