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HAAR-WAVELET METHOD FOR DAMAGE DETECTION IN COMPOSITE STRUCTURES

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In recent years, wavelet analysis has emerged as a powerful mathematical tool for analyzing non-stationary signals, offering superior time-frequency localization compared to traditional Fourier analysis. This characteristic makes wavelet transforms particularly well-suited for processing signals obtained from structural health monitoring (SHM) systems, where damage often manifests as localized changes in the dynamic response of a structure. Among the various wavelet families, Haar-wavelets stand out due to their simplicity, computational efficiency, and ability to detect abrupt changes or discontinuities in signals. Their orthogonal and compact support properties make them an attractive choice for identifying localized damage events.

The application of wavelet analysis for damage detection in composite structures has gained considerable traction. Researchers have explored various wavelet types to analyze vibration responses, acoustic signals, and strain measurements, aiming to pinpoint damage locations and assess their severity. Specifically, the utility of Haar -wavelet analysis in polymers and composite materials has been demonstrated in several studies [1]. The discrete nature of Haar-wavelets allows for efficient decomposition of signals into different resolution levels, enabling the identification of singularities associated with damage. This method is particularly adept at highlighting localized stiffness reductions or mass changes caused by damage, which appear as high-frequency components in the wavelet domain. By analyzing the wavelet coefficients at different scales, it is possible to differentiate between healthy and damaged states and even characterize the type and extent of damage [2]. This introductory section will lay the groundwork for a detailed exploration of the Haar-wavelet method's theoretical underpinnings and its practical application for damage detection in composite structures.

In this research, damage detection is achieved by utilizing the energy of the sixth dyadic scale coefficients for the Haar-wavelet basis function. The selection of these particular scales is directly related to the support of the scaled wavelet basis. Higher scales correspond to a larger wavelet basis support, which in turn significantly increases the likelihood of detecting long-term changes or gradual structural degradation. Consequently, wavelet coefficients at these higher scales encapsulate crucial information about vibration modes at lower natural frequencies.

This methodology is specifically applied to the analysis of ambient vibration data. In such applications, damage detection typically focuses on the lower vibration modes. This is due to the inherent characteristic of ambient vibrations, where higher modes are unlikely to be sufficiently excited to provide reliable information for damage identification. Therefore, by concentrating on the energy within the sixth dyadic scale, this approach effectively targets the most relevant frequency ranges for detecting structural changes under typical ambient conditions. The numerical model for damage monitoring was based on the Haar-basis

$$W_H f(a,b) = [j/(2\pi a^{0.5})] \int_{-\infty}^{\infty} [F(s)/s] exp(jsb) \left[1 - exp\left(\frac{jas}{2}\right)\right]^2 ds \,, \quad (1)$$

where a, b are the dilated and scaled coefficients; $j = (-1)^{0.5}$.

The damage sensitivity function is derived for the composite system using the Haar-wavelet function

$$\Lambda_H = \left\{ \left[G(p)^2 \left(1 - 2c_1 cos\left(\frac{a\omega_d}{2}\right)\right)^2 + G(q)^2 (1 + c_2^2)^2 \right] / \left(4a(1 - \xi^2)\right) \right\}, \quad (2)$$

where $p, q = j\xi\omega_n \pm \omega_n (1 - \xi^2)^{0.5}$ *are the poles of damage contour integration;*
$G(m)$, $m = p, q$ *are the Fourier transforms of the forcing damage function;*
$c_1 = 1/c_2 = exp(-a\xi\omega_n/2)$ *are the damping coefficients;*
$\omega_d = \omega_n (1 - \xi^2)^{0.5}$ *are the damped natural frequencies;*
$\xi_{1,2} = c_{1,2}/(2\omega_n)$ *is the damping ratio.*

Damage detection operates on the premise that a damage-sensitive feature will change upon damage initiation. In this study, the damage-sensitive feature is defined as the energy of the sixth binary scale wavelet coefficients using the Haar wavelet. Damage is presumed to occur if a difference exists in the mean values of this feature. The sixth binary scale of the Haar-wavelet was chosen because it's optimal for capturing critical, damage-sensitive signal characteristics for this specific application. Generally, multiple scales should be investigated to identify the best candidate for damage detection.

Summary and conclusions. In this paper, a damage-sensitive feature based on the wavelet transform of the vibration signal is derived. The damage-sensitive feature is defined as the energy of the wavelet coefficients at large scales. The theoretical aspects of the wavelet decomposition of vibration signals are presented. Expressions for the energies of the wavelet coefficients are obtained using the Haar-wavelet basis function, namely, the Haar-wavelet based feature for mechanical damage. The set of features is obtained for a single-degree-of-freedom system and a multi-degree-of-freedom system. For a single-degree-of-freedom system, the damage-sensitive feature depends on the natural frequency of the composite system, the attenuation coefficient, and the scale a. For a system with many degrees of freedom, the damage-sensitive feature contains modal information of the system through the eigenvectors, eigenfrequencies, and damping coefficients of the system. The relationships for these coefficients are written assuming proportional damping; and a slight change in the scale of the Haar-wavelet basis.

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