SECTION XX. SYSTEM ANALYSIS, MODELING AND OPTIMIZATION

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APPLICATION OF CHARACTERIZATION ANALYSIS METHODS TO INVESTIGATION OF LOGICAL NETWORKS STRUCTURES

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The complication of data networks causes the problem of increasing the number of failures in the network, since reliability is a function of the number of network elements. One of the promising methods for optimizing the structure of control systems in networks is the method of characterization analysis.

Of greatest interest are the works under the guidance of Professor Gorbatov V.A. The results of his work have been published in a number of publications on the fundamentals of discrete mathematics, in applied works on CAD and logical control of distributed systems [1].

When creating models of distributed logic control systems, the mathematical apparatus of graph theory is used, and the problem posed is reduced to the problem of embedding graphs in other graphs that have special structural properties. The control system element model is represented as an initial graph *G*, and the properties that satisfy the object when performing transformations are determined by the conditions graph *R*.

An embedding of a graph *G* into a graph *R* is a mapping of their supports $f : V(G) \to V(R)$, under which the signatures $\forall_{i,j}(v_i, v_j) \in U(G) \Rightarrow \left(f(v_i), f(v_j)\right) \in U(R)$. In this case, the embedding problem reduces to extracting from the graph *R* a subgraph isomorphic to the graph *G* [2].

The search for an optimal solution according to some criterion requires transformations by a minimal extension or a minimal narrowing of the signature of the graph *G*. Graph embedding problems are logical-combinatorial in nature, and since the dimension of the graph *R* is large enough, it becomes impossible to obtain an optimal solution by enumeration of all options even when using high-speed equipment. To avoid such enumeration, the principle of characterization analysis based on the search for forbidden nesting figures is used.

The forbidden figure of the embedding of a graph *G* into a graph *R* is a subgraph *G′* of the graph *G* that is not embedded in *R*, when at least one element of the support (signature) is removed, under which it is embedded in *R*. The problem of finding forbidden figures is called characterization. To solve the problem, it is required to construct a set of forbidden figures and determine the minimum number of signature elements of the graph *G* that prevent embedding in the graph *R*. Analysis of the influence of forbidden embedding figures on the properties of the support and signatures of the graph *R* in solving a specific problem makes it possible to optimally implement the embedding $G \to R$ without extracting and removing forbidden figures, by finding signatures of the graph *G* that prevent the necessary transformations from being prevented [3].

The solution of a specific characterization problem of embedding $G \to R$ consists of the following main steps:

1) set's construction the forbidden figures of embedding $G \to R$ using procedures based on homomorphism;

2) highlighting the analysis of the forbidden figures influence on the properties of the support and the signature of the graph *R* at the level of the support's elements and the signature of the graph *G*;

3) formation of the embedding method $G \to R$ in a minimal way based on consideration of the semantics the transformations conducted;

4) the implementation the semantic method of large dimension's (more than 100 vertices) embedding graphs in an optimal way based on the graph differentiation apparatus [1, 4].

To study the logical networks structure and solve the characterization problem, we apply the concept of a derivative based on the use of the letters frequency in the a certain model's Ψ words.

Let a graph *G* be given, which contains five edges and four vertices, and the words of the model are the skeletons of the graph *G* (Fig. 1.)

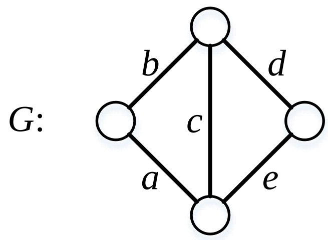

Fig. 1. **Graph *G***

Figure 2 shows eight skeletons, and the participation frequency of edges in the formation of the graph's skeleton G can be characterized by the occurrences number each of the edges in these skeletons (Fig. 2).

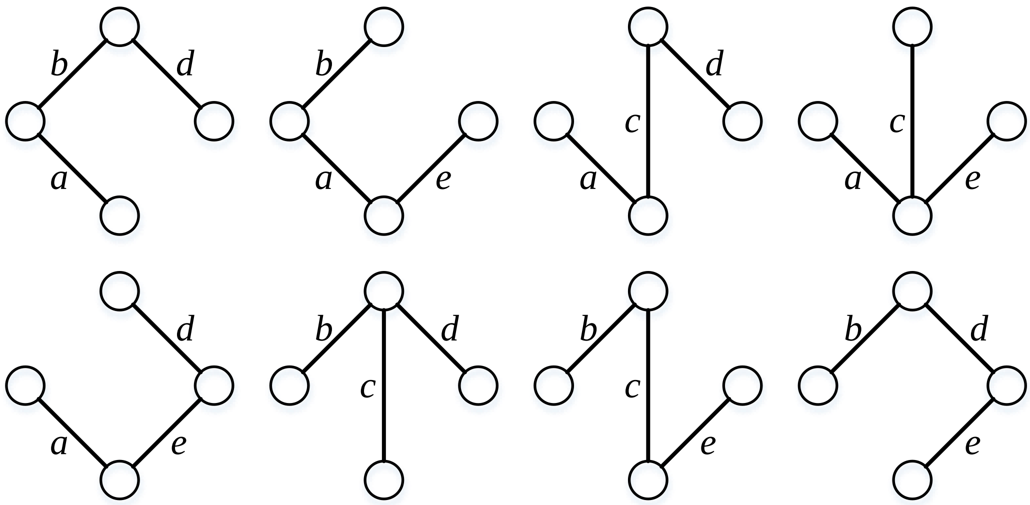

Fig. 2. **Graph's skeletons *G***

Thus, edge *a* participates 5 times in the formation of skeletons, edge *c* – 4 times, and so on. Then the desired frequency of a pair of edges ρ_i and ρ_j will be determined by the ratio of the skeletons number that contain the edge ρ_i or ρ_j, but do not contain them at the same time, to the number of skeletons containing both the edge ρ_i and the edge ρ_j: $(f_i - 2f_{ij} + f_j)/f_{ij}$, where f_i, f_j, f_{ij} – the number the graph's skeletons, which included the edges ρ_i, ρ_j, ρ_i and ρ_j simultaneously, respectively, and is the derivative of the graph *G* by the event *S* is the formation the graph's skeleton *G*, $\frac{\partial G}{\partial S}$.

This ratio shows the participation's non-uniformity degree of pairs edges in the formation the graph's skeletons.

Each event S defines some two-dimensional binary matrix $Q = [q_{ij}]_{m\times n}$, where each column mutually corresponds to a condition included in the event (letter of the model), and each row corresponds to a set of conditions (words of the model) under which the event takes place (true), and

$$q_{ij} = \begin{cases} 1, \text{if the jth condition is included in the ith set of conditions under which the event is true} \\ 0, \quad \text{otherwise} \end{cases}$$

The frequency matrix of relations $F = [f_{ij}]_{n\times n}$ is a matrix, each row (column) of which mutually corresponds to a letter, and the element f_{ij} is equal to the number of words that include letters *i* and *j*, if $i \neq j$, otherwise $(i = j)$ – the number of words that include the letter *i*. The relationship matrix $F = [f_{ij}]_{n\times n}$ is symmetrical with respect to the main diagonal, i.e. $f_{ij} = f_{ji}$ and the natural frequency of any letter is not less than the mutual frequency of this letter with any other letter.

The frequency matrix of relations F and the incidence matrix Q will satisfy the following relation (1):

$$\mathrm{F} = \mathrm{Q}^{\mathrm{T}} \times \mathrm{Q}, \qquad (1)$$

where:
Q^T *– transposed matrix.*

The degree the graph's components of heterogeneity *G* with respect to a given event S is characterized by the derivative $\partial G/\partial S$ of the graph *G* with respect to the event *S*.

The derivative $\partial G/\partial S$ of a graph *G* with respect to an event *S* is an undirected weighted graph $\langle V,(U,P)\rangle$, whose support coincides with the support of the model determined by this event, and the pair of vertices (v_i, v_j) is weighted by the frequency ratio $(f_i - f_{ij}) + (f_i - f_{ij})$ of their non-joint participation to the frequency f_{ij} of joint participation in the event *S* (2):

$$\frac{\partial \mathrm{G}}{\partial \mathrm{S}}(\mathrm{v_i}, \mathrm{v_j}) = \frac{\mathrm{f_i} - 2\mathrm{f_{ij}} + \mathrm{f_j}}{\mathrm{f_{ij}}} \qquad (2)$$

where:
$(v_i, v_j) \notin U$, *if* $\frac{\partial G}{\partial S}(v_i, v_j) = \infty$;
$(v_i, v_j) \in U$, *if* $\frac{\partial G}{\partial S}(v_i, v_j)$ *– final value other than zero;*
$v_i = v_j$, *if* $\frac{\partial G}{\partial S}(v_i, v_j) = 0$.

Using formulas (1) and (2), we find the graph's derivative *G* with respect to the event *S*, which will characterize the intensity of the edges' participation in the formation the graph's skeletons G.

The incidence matrix of the model's event has the form:

$$Q = \left\| \begin{matrix} \mathrm{a} & \mathrm{b} & \mathrm{c} & \mathrm{d} & \mathrm{e} \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{matrix} \right\|. \qquad (3)$$

In this matrix, each column corresponds to an edge of the graph G (a condition included in the event), a row corresponds to a set of edges that form the skeleton of the graph (a set of conditions under which a given event occurs) (Fig. 2).

The frequency matrix of relations F, corresponding to the matrix Q:

$$F = Q^T \times Q = \begin{matrix} & a & b & c & d & e & \\ & 5 & 2 & 2 & 3 & 3 & a \\ & 2 & 5 & 2 & 3 & 3 & b \\ & 2 & 2 & 4 & 2 & 2 & c \\ & 3 & 3 & 2 & 5 & 2 & d \\ & 3 & 3 & 2 & 2 & 5 & e \end{matrix}$$

Let us calculate the value of the derivative on the graph's edges depicted in Fig.1:

$$\frac{\partial G}{\partial S}(a,b) = \frac{f_a - 2f_{ab} + f_b}{f_{ab}} = \frac{5 - 2\cdot 2 + 5}{2} = 3,$$

$$\frac{\partial G}{\partial S}(a,c) = \frac{f_a - 2f_{ac} + f_c}{f_{ac}} = \frac{5 - 2\cdot 2 + 4}{2} = 2{,}5,$$

$$\dots \; \dots \; \dots \; \dots \; \dots$$

$$\frac{\partial G}{\partial S}(d,e) = \frac{f_d - 2f_{de} + f_e}{f_{de}} = \frac{5 - 2\cdot 2 + 5}{2} = 3.$$

We use the resulting values to build a graph $\frac{\partial G}{\partial S}$.

We get the graph $\frac{\partial G}{\partial S}$ depicted in fig. 3

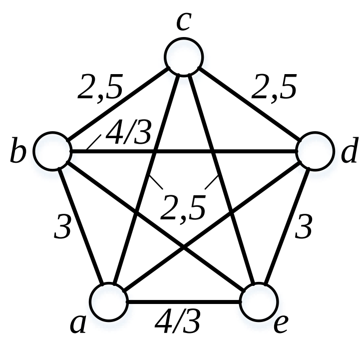

Fig. 3. **Graph** $\frac{\partial G}{\partial S}$

The differentiation method can also be used for more complex networks with the largest number of parameters and vertices [5].

Of the above methods, to solve the characterization problem, the event differentiation method was applied to determine the frequency of participation of edges in the formation the graph's skeletons G.

This method can be used to create models of the structure of control systems for distributed systems and, by solving graph embedding problems, it is possible to determine the conditions that satisfy the specified requirements for the system being designed.

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